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Embracing sum using Ascent Block of Γ_1-non deranged permutation

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ABSTRACT
In this paper, some combinatorial properties of the scheme called Γ_1-non deranged permutations, especially in relation to right and left embracing sum using ascent block were identified and studied. This was done first through some computation on this scheme using prime numbers $p \geq 5$ .In this work, we redefine the right embracing sum and left embracing sum on Γ_1- non deranged permutations. We also observed that the right embracing sum $\mathrm{Res}(\omega_i)$ is equal to the right embracing sum $\mathrm{Res}(\omega_{p-i})$ where $1 \geq i \leq p-1$,$\mathrm{Res}(\omega_i) = \frac{(p-3)(p-1)}{8}$ where $i = \frac{p-1}{2}, \frac{p+1}{2}$ Similarly, the left embracing sum $\mathrm{Les}(\omega_i)$ is equal to the right embracing sum $\mathrm{Res}(\omega_i)$ that is $\mathrm{Les}(\omega_i) = \mathrm{Res}(\omega_i) = 0$, $\mathrm{Les}(\omega_{\frac{p+1}{2}}) = \frac{p^2-1}{8}$ where $p \geq 5$ other properties were also observed.

KEY WORDS: Ascent Block, right embracing number, left embracing number and Γ_1- Non Deranged Permutations

I. INTRODUCTION

The study of permutation statistics has been a trending area of research by many authors due to its application in engineering and science, it deals with permutations whose element are numbers, in which it will examined the arrangement of the permutations. The origin of permutation statistics was coined to [1] who was the first to defined permutation statistics for a permutation $\pi \in S_n$, the number of descent (desπ), the number of excedance (excπ), the number of inversion (invπ) and the major index (majπ). He showed that the excπ is equidistributed with desπ and invπ is equidistributed with majπ over symmetric group S_n .Permutation statistics are of two major types; Eulerian statistics and Mahonian statistics where the Eulerian statistic is any statistic that is equidistributed with the des and the Mahonian statistics is any statistics that is equidistributed with the inv. The Aunu permutation pattern first arose, out of attempts to provide some combinatorial interpretations of some succession scheme and today series of results ranging from its avoinding class to its properties which have also been studied. More recently the Catalan numbers were used by [2] to develop the scheme for prime number $p \geq 5$ and $\Omega \subseteq N$ which generate the cycle of permutation patterns using $\omega_i = ((1)(1+i)_{mp}(1+2i)_{mp} \ldots (1+(p-1)i)_{mp})$ to determine the arrangements. This permutation pattern was further studied by [3] to establish a permutation group, they achieved this by embedding an identity element $\{1\}$ in the collection of $\omega_i = ((1)(1+i)_{mp}(1+2i)_{mp} \ldots (1+(p-1)i)_{mp})$ besides the general representations of groups and symmetric group. Researchers have over time looked also at permutation group with certain properties; one that comes to mind is the permutation patterns that have any of the elements fixed or the one that has no fixed element, here the idea of deranged and non-deranged permutation surface. It is in line with this understanding that [3] used the Aunu permutation of [2] to a two line notation and to establish a different permutation group (Γ_1-non deranged permutation group $\mathcal{G}_p^{\Gamma_1}$). This enabled them a further study on the theoretical and algebraic properties of permutation group using this two line structures. Besides the representation of this permutation group $\mathcal{G}_p^{\Gamma_1}$ as a $F\mathcal{G}_P$-module revealed some interesting results about the character of each element of $\mathcal{G}_p^{\Gamma_1}$. Also [4] introduced a special class of non-deranged permutation group $\mathcal{G}_p^{\Gamma_1}$ with fix at 1 and established the non normality of $\mathcal{G}_p^{\Gamma_1}$ as subgroup of S_n even when some subgroup of $\mathcal{G}_p^{\Gamma_1}$ are normal in$\mathcal{G}_p^{\Gamma_1}$. [5] described the representation of Γ_1-non deranged permutation group via the Young tableaux and establish that every Young tableuax of this permutation group is not standard.

[6] studied pattern popularity in Γ_1-non deranged permutations it establish algebraically that pattern τ_1 is the most popular and pattern τ_3,τ_4 and τ_5 are equipopular in $\mathcal{G}_p^{\Gamma_1}$ they further provided efficient algorithms and some results on popularity of patterns of length-3 in$\mathcal{G}_p^{\Gamma_1}$. [7] introduced fuzziness on Γ_1 -non deranged permutation group $\mathcal{G}_p^{\Gamma_1}$ and discovered that it is a one sided fuzzy ideal (only right fuzzy but not left) also the α-level cut of f coincides with if $\alpha = \frac{1}{p}$. [8] describe theoretical properties of Γ_1 -non deranged permutation in relation to the Ascent and shown that the union of the Ascent set is equal to the identity, also observed that the difference between $\mathrm{Asc}(\omega_i)$ and $\mathrm{Asc}(\omega_{p-i})$ is one. [9] provide very useful theoretical properties of Γ_1 -non deranged permutation in relation to the Excedance and shown that the Excedance set of all ω_i in $\mathcal{G}_p^{\Gamma_1}$ such that $\omega_i = e$ is $\frac{1}{2}(p-1)$. [10] applied the direct and skew sum operation on the elements of Γ_1-non deranged permutation group and presented the relations and schemes on the structures and fixed point of the permutations obtained from these operations. Further more, if π is the direct sum of these Γ_1-non deranged permutations, then the collection of permutations in the form of π is abelian group under composition denoted as $\mathcal{G}_p^{\Gamma_m\oplus}$. It also established that an isomorphism between $\mathcal{G}_p^{\Gamma_1} \times \mathcal{G}_p^{\Gamma_1}$ and $\mathcal{G}_p^{\Gamma_m\oplus}$ [11] showed that right embracing number $\mathrm{Res}(\omega_i)$ of Γ_1-non deranged permutations is equidistributed with the left embracing number $\mathrm{Les}(\omega_i)$ and the $\mathrm{Res}(\omega_i)$ is equidistributed with the $\mathrm{Res}(\omega_{p-i})$. It also observe that the height of the weighted motzkin path of ω_i is the same as the height of weighted motzkin path $\omega_{p-\mathrm{des}(\omega_i)}$. [12] established that the intersection of Descent set of all Γ_1 -non derangement is empty, also observed that the Descent number is strictly less than Ascent number by $p-1$. [13] Proved that inversion number and major index are not equidistributed in Γ_1 - non deranged permutations and also established that the difference between sum of the major index and sum of the Inversion number is equal to the sum of descent number in Γ_1 - non deranged permutations. [14] generated recursion formulas for maximum number of block and minimum number of block in Γ_1 - non deranged permutations and it also observed that $\mathrm{arc}(\omega_i)$ is equidistributed with $\mathrm{asc}(\omega_i)$ in Γ_1 - non deranged permutations. [15] proved that the statistics dez (cardinality of the set Dez)is an Eulerian statistics on Γ_1 - non deranged permutations due to its equidistributed with des(cardinality of the descent set), it also showed that the statistics maz (sum of the descent of ZDer) is equidistributed with maj (sum of the descent set). [16] studied ascent and descent blocks of Γ_1 - non deranged permutations. moreover, it defined the mapping $\Psi_{AI}: \mathcal{G}_p^{\Gamma_m}. \to \Omega_p$ which takes permutation from Γ_1- non deranged permutations group $\mathcal{G}_p^{\Gamma_m}$ to weighted Motzkin path in Ω_p with respect to ascent and descent blocks. it also investigated that Motzkin polynomial of $\mathcal{G}_p^{\Gamma_m}$ [17] studied permutation graph on Γ_1 - non deranged permutations using the set of inversion as edge set and the values of permutation as the set of vertices.From the graph it observed that radius of the graph of any ω_1 is zero, the graph of any $\omega_1 \in \mathcal{G}_p^{\Gamma_1}$ is null,and by restricting 1, the graph of ω_{p-1} is complete. More recently [18] established that the admissible inversion descent $\mathrm{aid}(\omega_{p-1})$ is equi-distributed with descent number $\mathrm{des}(\omega_{p-1})$ and also showed that the admissible inversion set $\mathrm{Ai}(\omega_i)$ and admissible inversion set $\mathrm{Ai}(\omega_{p-i})$ are disjoint.

In this paper, we show that the right embracing sum $\mathrm{Res}(\omega_i)$ is equal to the right embracing sum $\mathrm{Res}(\omega_{p-i})$ where $1 \geq i \leq p-1, \mathrm{Res}(\omega_i) = \frac{(p-3)(p-1)}{8}$ where $i = \frac{p-1}{2}, \frac{p+1}{2}$ and also show that the left embracing sum $\mathrm{Les}(\omega_i)$ is equal to the right embracing sum $\mathrm{Res}(\omega_i)$ that is $\mathrm{Les}(\omega_i) = \mathrm{Res}(\omega_i) = 0$, $\mathrm{Les}(\omega_{\frac{p+1}{2}}) = \frac{p^2-1}{8}$ where $p \geq 5$ other properties were also observe.

II. PRELIMINARIES

In this section, we give some definitions and preliminaries which are useful in our work.

Definition 2.1

Let Γ be a non empty set of prime cardinality greater or equal to 5 such that

$$\omega_i = \begin{pmatrix} 1 & 2 & 3 & \cdots & p \\ 1 & (1+i)_{mp} & (1+2i)_{mp} & \cdots & (1+(p-1)i)_{mp} \end{pmatrix}$$

is called the Γ_1-non deranged permutation. We denote $\mathcal{G}_p$ to be the set of Γ_1-non deranged permutations. The pair $\mathcal{G}_p$ and the natural permutation composition forms a group called the Γ_1-non deranged permutation group, denoted as $\mathcal{G}_p^{\Gamma_1}$. This is a special permutation group whose members fix the first element of Γ.

Definition 2.2

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An ascent of a permutation $f = \begin{pmatrix} 1 & 2 & . & . & . & n \\ f(1) & f(2) & . & . & . & f(n) \end{pmatrix}$ is any position $i < n$ (where i and n are positive integers and the current value is less than the next), that is i is a ascent of a permutation f if $f(i) < f(i+1)$. The ascent set of f, denoted $Asc(f)$, is given by $Asc(f) = \{i: f(i) < f(i+1)\}$. The ascent number of f, denoted by $asc(f)$, is defined as the number of ascents and is given by $asc(f) = |Asc(f)|$.

Definition 2.3
An ascent block of a permutation $\omega_i = a_1, a_2 \ldots a_n$ is the subword obtained by putting slash between a_i and a_{i+1} whenever $a_i > a_{i+1}$.

Definition 2.4
A proper ascent block is a ascent block that has more than one letter.

Definition 2.5
A improper ascent block is a ascent block that has one letter.

Definition 2.6
An opener of a permutation is the set smallest letter in a ascent block. For a permutation ω_i it is donated by $O(\omega_i)$, and its cardinality is denoted as $o(\omega_i)$.

Definition 2.7
A closer of a permutation is the set biggest letter in a ascent block. For a permutation ω_i it is donated by $C(\omega_i)$ and it is cardinality is denoted as $c(\omega_i)$.

Definition 2.8
Let B be a proper ascent block ofω_i. Then B is said to embrace letter a of ω_i if $c(B) > a > o(B)$.

Definition 2.9
The right embracing numbers of a permutation $\omega_i = a_1, a_2 \ldots a_n$ are the numbers $e_1, e_2, \ldots, e_n$, where e_i is the number of descent blocks in ω_i that are strictly to the right of a_i and that embraced a_i. The right embracing sum of ω_i, donated as $Res(\omega_i)$ is defined by
$Res(\omega_i) = e_1 + e_2 + \cdots + e_n$.

Definition 2.10
The left embracing numbers of a permutation $\omega_i = a_1, a_2 \ldots a_n$ are the numbers $e_1, e_2, \ldots, e_n$, where e_i is the number of descent blocks in $\omega_i$that are strictly to the left of a_i and that embraced a_i. The left embracing sum of ω_i, donated as $Les(\omega_i)$ is defined by
$Les(\omega_i) = e_1 + e_2 + \cdots + e_n$.

III. MAIN RESULTS

In this section, we present some results on right embracing sum and left embracing sum using ascent block of subgroup $\mathcal{G}_p^{\Gamma_1}$ of S_p (Symmetry group of prime order with ≥ 5).

Proposition 3.1
Let $\omega_i \in \mathcal{G}_p^{\Gamma_1}$. Then the

$$Res(\omega_1) = Res(\omega_{p-1}) = 0$$

Proof
The block of ω_1 is in the form $\omega_1 = 12345 \ldots p$, it is trivial that it is embracing number is zero, since ω_1 has no block. For $\omega_{p-1} = 1p - (p-1) - (p-2) - \cdots - 2$ only the first block is proper, so no letter in it can be embrace to the right, since all the blocks to its right are improper. Therefore, $Res(\omega_{p-1}) = 0$. Hence

$$Res(\omega_1) = Res(\omega_{p-1}) = 0.$$

Proposition 3.2
Let $\omega_i \in \mathcal{G}_p^{\Gamma_1}$. Then the

$$Res(\omega_i) = Res(\omega_{p-i})$$

Proof
Suppose $\omega_i = a_1 a_2 \ldots a_p$, then $\omega_{p-i} = a_1 a_p a_{p-1} \ldots a_2$, by relaxing 1 since it is the least then the $Res(\omega_i) = Les(\omega_i)$, since 1 has no effect in the right embracing number then $Les(\omega_i) = Res(\omega_{p-i})$. This implies that

$$Res(\omega_i) = Res(\omega_{p-i})$$

Proposition 3.3
Let $\omega_i \in \mathcal{G}_p^{\Gamma_1}$, where $i = \frac{p-1}{2}, \frac{p+1}{2}$. Then the $Res(\omega_i) = \frac{(p-3)(p-1)}{8}$.

Proof
Let the $Res(\omega_{\frac{p+1}{2}})$ with respect to $p \geq 5$ be T_p, it is in the form

$$\begin{aligned} T_5 &= 1 \\ T_7 &= 3 \\ T_{11} &= 10 \\ \vdots &= \vdots \end{aligned}$$

multiplying above equations by 2 we have

$$\begin{aligned} 2T_5 &= 2 \\ 2T_7 &= 6 \\ 2T_{11} &= 20 \\ \vdots &= \vdots \end{aligned}$$

we can rewrite this as

$$\begin{aligned} 2T_5 &= \left(\frac{5-3}{2}\right)\left(\frac{5-1}{2}\right) \\ 2T_7 &= \left(\frac{7-3}{2}\right)\left(\frac{7-1}{2}\right) \\ 2T_{11} &= \left(\frac{11-3}{2}\right)\left(\frac{11-1}{2}\right) \\ \vdots &= \vdots \\ 2Res(\omega_{\frac{p+1}{2}}) = 2T_p &= \left(\frac{p-3}{2}\right)\left(\frac{p-1}{2}\right) \end{aligned}$$

This implies that

$$\begin{aligned} 2Res(\omega_{\frac{p+1}{2}}) &= \left(\frac{p-3}{2}\right)\left(\frac{p-1}{2}\right) \\ Res(\omega_{\frac{p+1}{2}}) &= \left(\frac{p-3}{2}\right)\left(\frac{p-1}{2}\right)\frac{1}{2} \\ Res(\omega_{\frac{p+1}{2}}) &= \frac{(p-3)(p-1)}{8} \end{aligned}$$

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Since $Res(\omega_i) = Res(\omega_{p-i})$ then

$$Res(\omega_{\frac{p+1}{2}}) = Res(\omega_{p-\frac{p+1}{2}}) = Res(\omega_{\frac{p-1}{2}})$$

Proposition 3.4

Let $\omega_i \in \mathcal{G}_p^{\Gamma_1}$, where $\omega_i = e$. Then

$$Res(\omega_i) = Les(\omega_i) = 0.$$

Proof

Let $\omega_i = e$, then $\omega_i = 123\cdots p$ with only one block. Therefore, it has neither right embracing number nor left embracing number. That is

$$Res(\omega_i) = Les(\omega_i) = 0.$$

Proposition 3.5

Let $\omega_i \in \mathcal{G}_p^{\Gamma_1}$. Then the

$$Les(\omega_{p-1}) = p - 2$$

Proof

$\omega_{p-1} = 1p - (p-1) - (p-2) - \cdots - 2$ it contains only one proper block at first position, which it is trivial that its letter cannot be embrace and other letters can be embrace only in the proper block $1\,p$ since 1 is the least and p is the largest. Therefore,

$$Les(\omega_{p-1}) = p - 2$$

Proposition 3.6

Let $\omega_i \in \mathcal{G}_p^{\Gamma_1}$, where $i = \frac{p+1}{2}$. Then the

$$Les(\omega_i) = \frac{p^2-1}{8}.$$

Proof

Let the $Les(\omega_{\frac{p+1}{2}})$ with respect to $p \geq 5$ be T_p, it is in the form

$$\begin{aligned} T_5 &= 3 \\ T_7 &= 6 \\ T_{11} &= 15 \\ \vdots &= \vdots \end{aligned}$$

multiplying above equations by 2 we have

$$\begin{aligned} 2T_5 &= 6 \\ 2T_7 &= 12 \\ 2T_{11} &= 30 \\ \vdots &= \vdots \end{aligned}$$

we can rewrite this as

$$\begin{aligned} 2T_5 &= \left(\frac{5^2-1}{4}\right) \\ 2T_7 &= \left(\frac{7^2-1}{4}\right) \\ 2T_{11} &= \left(\frac{11^2-1}{4}\right) \\ \vdots &= \vdots \\ 2Les(\omega_{\frac{p+1}{2}}) = 2T_p &= \left(\frac{p^2-1}{4}\right) \end{aligned}$$

This implies that

$$\begin{aligned} Les(\omega_{\frac{p+1}{2}}) &= \left(\frac{p^2-1}{4}\right)\frac{1}{2} \\ Les(\omega_{\frac{p+1}{2}}) &= \frac{p^2-1}{8} \end{aligned}$$

Proposition 3.7

Let $\omega_i \in \mathcal{G}_p^{\Gamma_1}$, where $i = \frac{p-1}{2}$. Then the

$$Les(\omega_i) = \frac{p^2-9}{8}.$$

Proof

Let the $Les(\omega_{\frac{p-1}{2}})$ with respect to $p \geq 5$ be T_p, it is in the form

$$\begin{aligned} T_5 &= 2 \\ T_7 &= 5 \\ T_{11} &= 14 \\ \vdots &= \vdots \end{aligned}$$

we can see the difference between this sequence and the sequence in proposition 3.6 is 1 therefore

$$\begin{aligned} Les(\omega_{\frac{p-1}{2}}) &= \left(\frac{p^2-1}{8}\right) - 1 \\ &= \frac{p^2-1}{8} - 1 \\ &= \frac{p^2-1-8}{8} \\ &= \frac{p^2-9}{8} \end{aligned}$$

Proposition 3.8

Let $\omega_i \in \mathcal{G}_p^{\Gamma_1}$ Then the

$$Les(\omega_{\frac{p-1}{2}}) = p - 2.$$

Proof

Let the $Les(\omega_{\frac{p-1}{2}})$ with respect to $p \geq 5$ be T_p, it is seen th

$$\begin{aligned} T_5 &= 3 \\ T_7 &= 5 \\ T_{11} &= 9 \\ \vdots &= \vdots \end{aligned}$$

multiplying above equations by 2 we have

$$\begin{aligned} 2T_5 &= 6 \\ 2T_7 &= 10 \\ 2T_{11} &= 18 \\ \vdots &= \vdots \end{aligned}$$

we can rewrite this as

$$\begin{aligned} 2T_5 &= 2(5-2) \\ 2T_7 &= 2(7-2) \\ 2T_{11} &= 2(11-2) \\ \vdots &= \vdots \\ 2Les(\omega_{\frac{p-1}{2}}) = 2T_p &= 2(p-2 \end{aligned}$$

This implies that

$$2Les(\omega_{\frac{p-1}{2}}) = 2(p-2)$$
$$Les(\omega_{\frac{p-1}{2}}) = p-2$$

Proposition 3.9
Let $\omega_i \in \mathcal{G}_p^{\Gamma_1}$ Then the
$$Les(\omega_{\frac{p+1}{2}}) = \frac{p^2-1}{8}.$$

Proof
Let the $Les(\omega_{\frac{p+1}{2}})$ with respect to $p \geq 5$ be T_p, it is seen that

$$\begin{aligned} T_5 &= 6 \\ T_7 &= 15 \\ T_{11} &= 21 \\ \vdots &= \vdots \end{aligned}$$

multiplying above equations by 2 we have

$$\begin{aligned} 2T_5 &= 12 \\ 2T_7 &= 30 \\ 2T_{11} &= 42 \\ \vdots &= \vdots \end{aligned}$$

we can rewrite this as

$$\begin{aligned} 2T_5 &= \left(\frac{5^2-1}{8}\right) \\ 2T_7 &= \left(\frac{7^2-1}{8}\right) \\ 2T_{11} &= \left(\frac{11^2-1}{8}\right) \\ \vdots &= \vdots \\ 2Les(\omega_{\frac{p+1}{2}}) = 2T_p &= 2\left(\frac{p^2-1}{8}\right) \end{aligned}$$

This implies that

$$\begin{aligned} 2Les(\omega_{\frac{p+1}{2}}) &= 2\left(\frac{p^2-1}{8}\right) \\ Les(\omega_{\frac{p+1}{2}}) &= \left(\frac{p^2-1}{8}\right) \end{aligned}$$

IV. CONCLUSION

This paper has provided very useful Combinatorial properties of this scheme called Γ_1 -non deranged permutations in relation to right and left embracing sum using ascent blocks. We have shown that the right embracing sum $Res(\omega_i)$ is equal to the right embracing sum $Res(\omega_{p-i})$ where $1 \geq i \leq p-1, Res(\omega_i) = \frac{(p-3)(p-1)}{8}$ where $i = \frac{p-1}{2}, \frac{p+1}{2}$ and also shown that the left embracing sum $Les(\omega_i)$ is equal to the right embracing sum $Res(\omega_i)$ that is $Les(\omega_i) = Res(\omega_i) = 0$, $Les(\omega_{\frac{p+1}{2}}) = \frac{p^2-1}{8}$ where $p \geq 5$.

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